

A LINEAR PROGRAMMING MODEL FOR BEEF CATTLE PRODUCTION

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A linear programming model is defined in detail to describe an on-farm integrated beef production enterprise. The model includes cropping, feeding and breeding activities with their requirements for land, labor, animal housing and crop storage facilities. Net energy and digestible protein equations are used prior to the model construction to determine the nutrient requirements for cows, replacement heifers, feedlot heifers and steers, given weights and growth rates in young animals proportional to mature cow weights. A method for identifying the distribution of animals across different age groups is also described. Three enterprises with purebred cows of different mature size, each under typical Ontario farm conditions, are analyzed and interpreted.

Les auteurs ont élaboré en détail un modèle de programmation linéaire pour décrire un atelier intégré de production de bovins de boucherie dans des conditions normales d'exploitation. Le modèle comprend les aspects de productions végétales, d'alimentation et de reproduction, avec leurs besoins respectifs de sol, de main-d'oeuvre, de bâtiments d'élevage et d'installations d'entreposage. Préalablement à la construction du modèle, des équations pour l'énergie nette et la protéine digestible sont utilisées pour déterminer les besoins nutritifs des vaches, des génisses de remplacement, et des génisses et des bouvillons en parc d'engraissement, les poids et les taux de croissance des jeunes sujets étant calculés en proportion du poids des vaches adultes. On décrit, en outre une méthode pour établir la répartition des bovins dans les divers groupes d'âge. Trois ateliers de production, utilisant des vaches de race différant par leur taille adulte, et tous représentant des conditions d'exploitation typiques pour l'Ontario, sont analysés et les résultats interprétés.

The simultaneous determination of economically optimal cropping, feeding and breeding programs within the confines of available resources could have a major impact on the profitability of an integrated beef enterprise. Optimal feeding programs for the feedlot phase of production have been considered by Brokken (1971a,b), McDonough (1971) and Scott and Broadbent (1972) using linear programming (LP). Optimal weights for buying and selling feedlot cattle have been considered in addition to optimal feeding programs by Meyer and Newett (1970) and Kennedy (1972) using dynamic program-

ming. The effects of choices of both feeding and breeding programs for an integrated cow herd and feedlot operation have been reported by Long and Fitzhugh (1970) and Cartwright et al. (1972) using LP. Several profit equations have also been used to examine the effects of breeds and prices on profitability.

There has not as yet been an extensive consideration either of the optimal combination of cropping, feeding and breeding programs or of the extent to which optimal programs might be changed by varying resource limitations. Such limitations could exist for finance, land, buildings, machinery and labor. The economic importance of such characteristics as calving ease, rates of gain and mature weights have not been examined

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within a complete production and financial framework. The comprehensive analysis of the interplay between these characteristics within such a framework is essential to provide objective guidelines to managers of integrated beef farms for choices of breeds and crossbreeding plans.

Many aspects of an LP model of this type were described by Leigh (1972). As an extension and modification of his thesis work, the purposes of this paper are to (1) describe the use of LP in beef production planning on an integrated farm unit, (2) describe a set of animal and crop production values including numbers and weights of animals of various ages, feed and labor requirements and crop yields, and (3) illustrate the application of the model with an example.

I. LINEAR PROGRAMMING FRAMEWORK

A. General Description

Linear programming is a technique for determining the optimal use of resources expressed in terms of a set of constraints, when there are numerous methods of proceeding towards a specified objective (or objective function), and when that objective can be quantified, for example, as maximization of gross margin. Gross margin is the difference between total revenue and total variable costs, variable costs being those that change with levels of production. In LP each farm enterprise is represented as an activity or set of activities; for example, steer feeding or barley growing. When there is more than one feasible method of management within an enterprise, each method is described as an activity. For example, two crop growing activities could be described, each with a different level of fertilizer application. The procedures of buying and selling commodities may also be considered as activities. Depending on the specification, LP may be used to maximize or minimize the objective function by choosing a suitable combination of activities, and associated levels for operating each chosen activity. The optimal solution is obtained by evaluating alternative production methods, using the technical coefficients (for example, yields, feed requirements) and the values of inputs and outputs of each activity, and by choosing for the

integrated beef production system those which give the largest gross margin obtainable within the resource limitations. The solution obtained is also subject to the requirement that no activity level may be negative. Analyzing beef production via an LP model then requires the specification of a set of limits on the supply of each resource and a list of technical coefficients expressing each activity in terms of the resources it uses.

In this paper, the LP solution is interpreted in an equilibrium situation within a single production period. The numbers of replacement heifers, feedlot heifers and steers are specified, all relative to the number of cows, in an integrated operation at equilibrium. Reproductive rates and disposals must be taken into account in such a way that the herd size as well as the number of animals in each livestock class remain constant from one time period to the next. Change from one such equilibrium state to another may be considered in the framework of a polyperiod model, but is not dealt with in this paper.

B. Assumptions

The assumptions of LP are stated by Heady and Candler (1958), and are restated here using examples from integrated beef production. These assumptions are often made in other research techniques such as partial budgeting in farm management or simulation in animal breeding.

i. **LINEARITY AND ADDITIVITY.** As its name implies, LP assumes linearity; the total input or output of any activity is proportional to the level of that activity used. For example, the price per kilogram of a slaughter steer is assumed constant whatever the number of steers sold. When it is known that certain relationships required in the LP are not linear, for example feed requirements for maintenance of mature cows at different weights, it is possible to either calculate in advance the technical coefficients for the particular levels of the activity considered, or subdivide the activity into several activities for an LP approximation to the non-linear situation.

Additivity refers to the requirement that different activities should be independent.

For example, the use of rotational cropping where a legume crop precedes a cereal crop in order that the yield of the latter may be increased would constitute a non-additive or interaction situation. These crops should not, therefore, be programmed as two separate activities, unless the same rotation is always used in the same proportions.

ii. **DIVISIBILITY.** It is assumed possible to use resources and to produce commodities in quantities that are fractional units. For instance, in the optimal solution, fractions of hectares of land may be allocated or fractions of animals may be grown. Although this may not be realistic, it is often not an important problem in interpreting an optimal LP solution. If necessary, integer programming techniques are available to deal with whole numbers of units. These are, however, more expensive to use because of the greater number of computations required for solution.

iii. **FINITENESS.** Only a limited number of activities are generally considered in an LP model partly because of data, time, cost and interpretation limitations. This is important because it affects the degree to which conclusions may be extrapolated from the optimal solution. Finiteness may be relevant at different modelling levels. For example, there may be limits placed on the number of crop or livestock activities and on the number of resources considered.

iv. **SINGLE-VALUE EXPECTATIONS.** Information is often lacking on various technical coefficients and these must therefore be estimated. Others must also be provided as single values, although continually changing with time. Caution is therefore required in the interpretation of results. Range analyses may be completed on the optimal solution to identify to which coefficients the solution is most sensitive. A related LP technique called parametric programming is also available and allows for successive changes in values of technical coefficients, constraints or objective function within the time period of a single run on the computer. The many solutions can then be compared.

C. The Nature of Information Required

Technical coefficients form the major part of the model. Market prices may be derived

from current or future values. Crop yields are generally taken from field data. Feed requirements for animals may be supplied from field data or from predictive equations (algorithms) based on experimental data.

The production activities are specified by the appropriate technical coefficients. For example, a weaned calf operation, a feedlot program with purchased feeds, or an integrated cow-calf feedlot operation using home grown feeds would be represented as different sets of activities using special technical coefficients. The specification must also state whether feeds are purchased, sold, or both and whether other livestock activities are introduced as complementary, supplementary or competitive to beef.

The type of production system must be identified as batch or continuous and the time period involved must be specified. A calendar of events within a single period must be established to describe, for example, dates for calving, weaning, selling stock, planting and harvesting crops. The first three dates mentioned here would affect feed requirements; the last three would affect cash flows or credit availability at the bank.

It is possible to specify feed requirements in various energy units: total digestible nutrients, digestible energy, metabolizable energy or net energy. The last of these is theoretically the most precise measurement of the energy requirements, although in practice its use may be subject to error, as indicated by Dickie et al. (1973) and Kromann (1973). Protein requirements may be entered as crude or digestible protein. There are many requirements for vitamins and minerals, although consideration of these may not be very critical from a planning viewpoint.

The crops grown will depend on the type of land available. For evaluating beef production programs, the number of crops may be small. In extension-oriented work, a more complete set of crops would need to be considered.

Labor requirements must also be defined for both animal and crop production. In certain cases, the labor requirements per unit of activity may decrease as the number of units increase; but within a restricted range of animal numbers or land area for

crops, a labor requirement per head or per hectare is a reasonable approximation. The work force may consist of the owner-operator and his family, or may include hired labor. Labor might be hired on a per hour per day or per month basis depending on availability. A price on the owner-operator's time may be built into the model, thus requiring his time to be purchased to reflect both his labor and management contributions.

D. The Nature of Resource Constraints

Maximum available labor and land must be identified by constraints in an LP framework. In addition to owner-operator labor and owned land, these constraints should include the hired labor and rented land. If it is decided to hire labor or rent land as a block requirement, then integer programming may be used and the unit defined as the block size. The working capital requirements may be handled as cash flow in the LP model, and may or may not be constrained. A constraint could be a banker's limit on borrowed working capital. Buildings may represent a constraint on crop storage capacity and on herd size. In the case of beef cattle, however, it is possible that profitable production from both cows and feedlot animals may be accomplished with very minimal shelter facilities. If minimal housing is to be considered, there is a need to ensure that the feeding system and its labor requirements correspond.

In some instances, energetic efficiency or some other measure of biological efficiency might replace gross margins as the objective. Although it is possible to maximize this objective function in a similar manner, it would still be necessary in practice to define a series of constraints to limit physical and financial resources.

II. STRUCTURE

A. Animal and Crop Activities

i. SCHEDULE OF EVENTS THROUGH THE YEAR. Three seasons were defined in the model: winter, 11 Nov.-1 Mar.; spring, 2 Mar.-10 May; and summer, 11 May-10 Nov. The management system was defined such that the calving date was on the last day of winter (1 Mar.) and weaning of calves was 7

mo later (1 Oct.). After weaning, all male calves entered the feedlot. Female calves were divided into two groups at this stage, those to be kept as replacement heifers and the remainder to enter the feedlot. Replacement heifers entered the breeding herd a year later at 19 mo. All cows and replacement heifers were bred by artificial insemination (AI) in June, with a second insemination in July where necessary, and pregnancy-checked in September. Culling cows and replacement heifers for poor performance, old age (cows only) or failure to conceive was carried out at the end of September. Cows with dead calves were sold in March to save feed.

ii. ANIMAL NUMBERS AND AGES. Four classes of stock were defined: cows, replacement heifers, feedlot heifers and steers. It was necessary to calculate the proportions of animals in the last three classes, relative to cows, before running the LP model in order to fix the proportions in the solution.

The proportion of feedlot steers was derived as $\frac{1}{2}v$, where v was the viability, or the number of calves weaned alive per cow in calf. In order to calculate the distribution of female calves between the feedlot and replacement heifers, the replacement rate was defined as the sum of an annual attrition rate (a_c) among all cows and a disposal rate for aged cows (y), each expressed relative to the total size of the cow herd. The proportion (p_r) required as heifer replacements at 7 mo of age was then given by

$$P_r = (a_c + y)/(1 - a_h)$$

where a_h was the attrition rate among heifers, relative to the number of heifers at 7 mo. The proportion of feedlot heifers, relative to the cow herd size, was then given by the formula $(\frac{1}{2}v - p_r)$.

The production year was defined for convenience from weaning in October to culling in September. In this framework, the value of a_h was given as the sum of the non-pregnancy rate (x_h) and the death rate (d_h) of heifers, each expressed relative to the number of replacement heifers alive at 7 mo of age. For cows, the value of a_c was given by the formula $(x_c + d_c + c)$ representing involuntary losses due to non-pregnancy (x_c) and cow deaths (d_c) and voluntary losses

due to culling (c) for poor performance or death of calf.

The maximal survival time was taken as eight parities, based on studies by Rogers (1972). Although the report of Long et al. (1972) suggested that the net returns of the whole enterprise were different for herds up to 8 or up to 11 parities, there was a difference of less than 1% in favor of the enterprise with older cows.

It was only necessary to divide the cow class into the eight parity (or age) groups for the purposes of calculating feed requirements and disposal weights. The proportions of cows in successive parity groups were based on an exponential decline in numbers, as used by Long and Fitzhugh (1970) in their simulated beef herd. For each of the first seven parity groups, the proportion of cows culled (or dead) at the end of the production year relative to those alive at the beginning of that year was $(x_c + d_c + c)$. For the last parity group, the proportion y of the total cow herd was culled for old age in addition, with

$$y = (a_c + y) (1 - x_c - d_c - c)^8 \\ (a_c + y) (1 - a_c)^8.$$

Given a_c , the equation may be solved for y ; then given a_h , p , may be determined. The division of a_c into three components is needed when heterosis is introduced to compare different crossing systems.

Subsequent to the report of Long and Fitzhugh (1970) where an exponential decline in cow numbers with age was assumed, Cartwright et al. (1972) used an attrition rate which varied with age, the probability of survival from one year to the next being highest at intermediate ages. Reports in dairy herds suggested that attrition rates increased as cows became older (Rendel and Robertson (1950), Specht and McGilliard (1960) and Miller et al. (1967)). In their dairy model, Pearson and Freeman (1973) used a rate that increased with age. Culling and attrition rates could be changed over ages if data from beef cattle so indicated.

The value of a_h assumed here for purebreds was 0.125, with $x_h = 0.10$ and $d_h = 0.025$. Cartwright et al. (1972) used a value of 0.05 for a_h (as $x_h = 0.02$ and $d_h = 0.03$).

Pearson and Freeman (1973) used a value between 0.12 and 0.14 for the relative loss of heifers between 6 mo of age and the time of recording successful pregnancies. The value of a_c assumed here for purebreds was 0.18. Long and Fitzhugh (1970) used an attrition rate of 0.11 and a replacement rate of 0.20, while Long et al. (1971) used a replacement rate of 0.15. The death loss of cows (d_c) was taken as 0.01 per year. Although Yang (1969) reported higher values of 0.0194 for all Canada and 0.0197 for Ontario for the most recent year of information then available (1966–67), those death rates included all beef and dairy stock over 12 mo of age. The present value of d_c refers only to beef cows over 18 mo of age. The Texas workers and Pearson and Freeman (1973) used a value of 0.02 for cows in their simulated beef and dairy herds. A death rate of 0.02 for feedlot animals was assumed in the present model.

iii. ANIMAL WEIGHTS AND GROWTH RATES.

There are two possible methods by which animal weights and growth rates for each class of stock may be defined. First, complete field data may be taken for each breed or cross. Alternatively, mature body weights may be entered for each breed or cross, and the weights and growth rates at young ages derived from these, on the assumption that weights and gains per day at given ages are proportional to cow size. The latter approach has been presented by Weinbach (1941) and Brody (1945). In the present model for purebred animals, weights for immature breeding cows were taken as 64, 81, 89 and 95% of mature cow weight at 19, 31, 43 and 55 mo of age (Brody 1945, pages 571–573). Growth rates over those 4 yr were assumed to be linear within each 12-mo period. It is interesting to note that the energy requirement for cows in their last year of growth is 2 to 3% larger than that of the mature cow in this model (section II A iv). Some of the problems associated with a proportionality approach have been illustrated by Taylor (1965, 1968) and Taylor and Fitzhugh (1971) in analyses of relationships between mature weight and time taken to mature. Essentially, the problem relates to the genetic correlation of mature

weights and time taken to mature. The present approach assumes that the time to maturity is the same for all breeds. The weight of cows sold for non-pregnancy was taken as the expected weight for their age; all other cows were considered to be disposed of at 90% of that expected weight.

Birth weights of female calves were taken as 6% of mature cow weight; a value of approximately 6.5% was used by Long et al. (1972 unpublished). Growth rates of feedlot heifers were assumed to be linear from birth, gaining at 0.175% of mature cow weight per day. A value averaging about 0.16% was used by Long et al. (1972 unpublished), where they described a considerable feed intake restriction on a silage-based diet for part of the growth period. Heifers were grown to a weight at constant finish representing 75% of mature cow weight, similar to the assumption made by Long et al. (1972 unpublished). From 7 mo of age, replacement heifers were fed a ration containing less energy and protein to be consistent with a linear growth rate to 64% of mature cow weight at 19 mo of age.

Birth weights of male calves were taken as 6.6% above those of females (Gregory et al. (1950), Burris and Blunn (1952), Guilbert and Gregory (1952), Touchberry and Bereskin (1966), Fahmy and Hidirolou (1970) and Preston and Willis (1970)). Steer calves were considered to grow at a rate of 11.1% above (Preston and Willis 1970), and to reach weights at constant finish of 11.5% above those of females (Preston and Willis 1970).

Carcass composition was assumed to be constant because of constant finish at slaughtering. No allowance was made for variability in slaughter criteria or carcass quality as measured by muscle to bone ratios or distribution of cuts. The effects of selection on growth rate and weight at constant finish would require technical coefficients provided by a new algorithm.

iv. **FEED INPUTS FOR CATTLE.** Feed requirements were computed separately for each of the four classes of stock. The cow class represented a weighted average of the requirements of cows in the four immature age groups and the four mature age groups.

The diet for mature cows was formulated with the assumption that there was no seasonal change in body weight. Since each class of stock had a growth component, an algorithm was developed to compute the daily requirements and to cumulate these requirements as the animals grew over each of the three seasons. The net energy system of Lofgreen and Garrett (1968) was found to be readily adapted for calculating the energy requirements based on body weights and growth rates, as follows:

$E_M = 0.077W^{0.75}$, where E_M is the net energy for maintenance for a day (Mcal), and W is body weight (kg).

$E_G = (0.05603G + 0.01265G^2)W^{0.75}$, where E_G is the net energy for gain in females for a day (Mcal), and G is the growth rate (kg/day). The coefficients in the E_G equation for steers are 0.05272 and 0.00684, respectively, for G and G^2 .

The use of energies for maintenance and gain was programmed in the model as described by Scott and Broadbent (1972).

The protein requirement was calculated using Preston's (1966) equation:

$D = 0.00279 (1 + 1.905G)W^{0.75}$ where D is the digestible protein requirement for a day (kg).

Extra requirements for pregnancy and lactation were then added to the cow values. Lactation was assumed to be independent of cow size (Long et al. 1971), and no allowance was made for age of cow effects on calf growth. The energy and protein requirements for early calf growth were added to the cow's spring and summer feed intake using the energy and protein equations as they applied to calf weights and calf growth rates. More detailed requirements have been provided by Long et al. (1972 unpublished) and Sturgess (1974). As in the Texas model, calf requirements in excess of those satisfied by the cow's milk were met with creep feed, above an arbitrary calf weight of 166 kg in this model. The creep feed was provided in the form of purchased rolled oats.

Pregnancy requirements were derived from the National Academy of Science-National Research Council (NAS-NRC 1970a) publication for daily dairy cow requirements, assuming that the extra feed

was required for the last 60 days of pregnancy; 3.8% of the annual net energy for maintenance and 5.3% of the annual digestible protein intake were added to the cows' winter feed intake to satisfy the total pregnancy requirement. The energy value was intermediate to the values used by Long and Fitzhugh (1970) and by Long et al. (1971). The protein value was 50% larger than that used by both sets of authors.

Barley, corn grain, corn silage and protein supplement were assumed to be available for the entire year for all classes of stock. In addition, hay was available in winter and spring, and pasture in summer, for cows and replacement heifers. Feedlot animals were given no access to pasture, but hay could be fed in any season. Intake limitations and their associations with body weight or some power of body weight have been discussed by several authors, for example Taylor (1971). In this model, cows and replacement heifers were allowed a maximal daily dry matter intake of 2% of body weight, and feedlot animals 2.5%, as described by Long et al. (1971).

The choices among these feedstuffs and the proportions of each chosen were estimated for each class of stock and season in the LP. The expected energy and protein densities for feedstuffs are given in Table 1. The optimal allocation of feedstuffs was the least-cost-mix, given the crop production and feed purchasing activities and the resource constraints in the model.

v. LABOR. The model assumed that the owner-operator was always available to work, whether required or not, and that he could provide up to 200 h per mo. In addition, labor hired on an hourly basis could be used at a rate of \$3/h, to a monthly limit defined at the time of running the LP. The cost of owner-operator labor was considered as fixed, and therefore could be charged as an expense after finding the LP solution to each example.

The efficiency of utilization of available labor depends on the level of farm organization and the motivation of the workers. The numbers of hours per hectare for grown crops and per head of cattle assumed here are shown in Table 2. Driver et al. (1974) have considered the relationship between

weather patterns (the likelihood of suitable weather) and labor for crops in their dairy farm planning model but this aspect is not developed here.

The monthly labor requirement from November to April was assumed to be 1.4 h/cow, with an additional 2 h/cow at the end of February or beginning of March for calving and 0.6 h/cow in April for attendance to the young calf. Monthly labor requirements for the remaining months were taken as 0.4 h/cow, using an additional 0.5 h/cow in May associated with turning out cows for summer pasture, 0.4 h/cow in June and in July for AI, with a second insemination where necessary, and 0.2 h/cow in September to check for pregnancy and to castrate male calves. Feedlot animal requirements were 0.35 h/animal for each month from November to April and 0.25 h/animal for each remaining month. Replacement heifers were assumed to require the same labor inputs as feedlot animals, except for an additional 0.5 h/heifer in May associated with turning out heifers to summer pasture, 0.4 h/heifer both in June and July for AI and 0.1 h/heifer in September to check for pregnancy.

vi. VARIABLE COSTS. Variable costs per hectare of grown crops and per head of cattle are shown in Table 3. For crops, these costs were considered to include seed, fertilizers and sprays purchased 1 mo before field use. The remainder of the annual variable cost for each crop was then equally divided between the 12 mo to account for the costs of machinery repairs, fuel and miscellanea.

For cattle, certain variable costs, which were per head costs and assumed constant across all breeds, were \$1.40/cow for each month from November to April and \$1.15/cow for each remaining month. These were considered to include the costs of bedding, general veterinary expenses, repairs, equipment use and miscellanea. To these were added \$2/cow for veterinary costs at calving, \$7/cow for AI in June and \$1/cow for veterinary costs associated with pregnancy-checking in September. Similarly, certain variable costs for feedlot animals and replacement heifers were taken as \$1.80/animal/mo. The bedding costs for replace-

Table 1. Technical coefficients for feedstuffs

Crop	Yield† (kg/ha)	Dry matter %	Losses‡ (as a % of crop remaining)			Source§ page, item	Digestible protein (% of dry matter)	Net energy (Mcal/ kg dry matter)		Variable// cost when grown (\$/ha)	Purchase¶ price (\$/kg)
			Harvesting	Storage	Feeding			Maintenance	Gain		
Barley	2,630	89	10	2	2	28,60	9.8	2.13	1.40	58.9	0.0471
Corn grain	4,760	89	10	2	2	32,61	7.5	2.28	1.48	115.2	0.0483
Corn silage	34,300	35††	5	13	2	32,35	4.7	1.56	0.99	123.0	—
Oats	—	89	—	2	2	38,28	9.9	1.73	1.14	—	0.0502
Hay	7,540‡‡	89††	17	2	3	28,24	6.6††	1.15	0.43	59.3	0.0220
Pasture	16,800‡‡	20††	—	—	—	44,34	12.1††	1.31	0.69	29.7	—
Protein supplement	—	89	—	—	2	42,61#	39.4	1.74	1.16	—	0.1670

†From 1972 Ontario averages in Ontario Ministry of Agriculture and Food (OMAF 1973) and Driver et al. (1974).

‡Adapted from Stonehouse (1971) and Driver et al. (1974).

§Dry matter yields, energy and protein densities in National Academy of Sciences (NAS-NRC 1970b).

//Adapted from OMAF (1969) and Driver et al. (1974).

¶Weighted monthly values for Ontario, 1972 in OMAF (1973).

#Energy and protein densities taken as 90% of NAS-NRC values; this allowed for addition of a mineral mix.

††Modified from NAS-NRC table (Mowat, personal communication).

‡‡Equivalent to a standing crop of 33.6 metric tons/ha (= 33,600 kg/ha).

Table 2. Labor requirements for crops† and animals‡

Item (h/ha or h/head)	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.	Total
Barley	—	—	—	1.23	2.47	0.49	—	5.43	1.48	2.22	0.25	—	13.56
Corn grain	—	0.25	—	0.49	3.46	1.23	0.25	0.25	—	3.21	3.95	0.49	13.56
Corn silage	—	—	0.99	—	3.70	1.48	—	—	5.43	5.68	0.99	0.25	18.53
Hay	—	0.74	—	0.49	0.37	4.45	4.45	1.98	—	0.25	—	—	12.73
Pasture	—	0.49	—	0.37	0.37	0.37	0.37	—	—	0.12	—	—	2.09
Cows	1.40	2.40	2.40	2.00	0.90	0.80	0.80	0.40	0.60	0.40	1.40	1.40	14.90
Replacement heifers	0.35	0.35	0.35	0.35	0.75	0.65	0.65	0.25	0.35	0.25	0.35	0.35	5.00
Feedlot heifers or steers	0.35	0.35	0.35	0.35	0.25	0.25	0.25	0.25	0.25	0.25	0.35	0.35	3.60

†Adapted from Ontario Ministry of Agriculture and Food (OMAF), (1969), values converted from hours/acre.

‡Adapted from OMAF (1969) and Lovring and MacMinn (1973).

ment heifers (\$0.30/head) were, however, subtracted for each month from May to October. An extra \$1/animal in October was added to account for veterinary costs associated with weaning, \$7/replacement heifer in June for AI, and \$1/replacement heifer in September for veterinary costs associated with pregnancy-checking.

Total variable costs also include feed, labor and interest costs, which are variable costs since they vary with production level. These were specifically considered through costs of crop production, purchased feed and purchased animals and were assessed directly in the LP. Total variable costs were then not constant per animal.

vii. CROP YIELDS. Table 1 gives expected values of yields from barley, corn grain, corn silage, hay and pasture as kg of wet material/ha, the dry matter percentage, and losses through harvesting, storage and feeding. For each crop, the four factors representing dry matter proportion and the proportions retained at each of the three handling stages were applied multiplicatively to the yield of wet material to determine the effective dry matter yield. Variable costs/ha are also given. From these costs, the effective cost/kg of dry matter could be derived.

viii. COMMODITY PRICES. For purchased feedstuffs, (barley, corn grain, hay, rolled oats for creep feed, and protein supplement), the cost of wet material is shown in Table 1. Values were taken as the 1972 weighted monthly means for Ontario (Ontario Ministry of Agriculture and Food (OMAF 1973)). Effective dry matter yields were computed by using the multiplicative factors described in section II A vii except that no figure for harvest loss was required.

Livestock prices (in \$/kg) were weighted monthly means for Toronto from the Canada Department of Agriculture (CDA 1973a) reports of 1972 prices. The values used were: the slaughter steers (0.791), slaughter heifers (0.734) and (1) good cows (0.586), (2) medium cows (0.553) and (3) 'cut-and-can' cows (0.456) for the disposal of cows culled (1) for non-pregnancy in September, (2) for poor performance in September or with dead calves in March, and (3) for old age in September, respectively. The disposal

value (0.711) of non-pregnant replacement heifers was taken as the value of good stocker heifers at Toronto (CDA 1973b). The salvage value of dead animals was \$4/head. All unit prices were assumed to be independent of weight.

ix. INTEREST PAYMENTS. A cash flow, or cumulative balance of income and expenses, was computed in the LP model for each of the 12 mo, beginning with a bank balance of zero in October. Payment of interest on working cash borrowed from the bank, and payment of interest by the bank on working cash surplus to requirements for operating the beef enterprise vary with time, with the banking company and with the circumstances of the farmer. Since the mean 1972 annual interest rate was 4.75% (Bank of Canada 1973) and commercial banks were able to buy money from the Bank of Canada at about 6%, compounded monthly interest rates of 0.565% and 0.407% were used in this model for borrowing and depositing working cash at the bank. These were equivalent to annual interest rates of 7.0 and 5.0%, respectively.

The valuation price of livestock (\$/kg) was included in the cash flow (but not in the objective function). The CDA (1973a) values for good cows (0.586), for good stocker steers (0.877) and the CDA (1973b) value for good stocker heifers for feedlot or replacement (0.711) were charged in October. Pearson and Freeman (1973) included interest on the valuation price of livestock, but applied this to cows only.

B. Gross Margin

The gross margin was defined as the difference between total revenue from meat (and from crops, if sold) and the sum of variable costs as in section II A vi, hired labor, hired land (at \$60/ha when available), purchased feed and net interest on working cash.

C. Fixed Costs

Fixed costs may be considered after completing the LP solution, to determine net returns. Fixed costs of an enterprise include depreciation on buildings, machinery and equipment, taxes, payment to the owner-operator for his labor and management, and interest payments other than for livestock.

Table 3. Variable costs for crops† and animals‡

Item (\$/ha or \$/head)	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.	Total
Barley	2.23	2.23	31.26	2.23	5.31	2.23	2.23	2.23	2.23	2.23	2.23	2.23	58.9
Corn grain	3.29	3.29	3.29	65.00	17.32	3.29	3.29	3.29	3.29	3.29	3.29	3.29	115.2
Corn silage	3.95	3.95	3.95	65.50	17.97	3.95	3.95	3.95	3.95	3.95	3.95	3.95	123.0
Hay	2.47	2.47	2.47	32.11	2.47	2.47	2.47	2.47	2.47	2.47	2.47	2.47	59.3
Pasture	0.82	0.82	0.82	20.66	0.82	0.82	0.82	0.82	0.82	0.82	0.82	0.82	29.7
Cows	1.40	2.40	2.40	1.40	1.15	8.15	1.15	1.15	2.15	1.15	1.40	1.40	25.3
Replacement heifers	1.80	1.80	1.80	1.80	1.50	8.50	1.50	1.50	2.50	2.50	1.80	1.80	30.6
Feedlot heifers or steers	1.80	1.80	1.80	1.80	1.80	1.80	1.80	1.80	1.80	2.80	1.80	1.80	22.6

† Adapted from Ontario Ministry of Agriculture and Food (OMAF 1969), and Driver et al. (1974), values converted from \$/acre.

‡ Adapted from OMAF (1969) and Lovering and MacMinn (1973).

The interest on capital for livestock was considered as a variable cost in the model. This assumes that capital required for livestock is always available, but that interest must be paid by the enterprise. Ranking of enterprises on the basis of farm gross margins would be the same as ranking on the basis of net returns above total farm costs for a farm with given fixed costs.

III. EXAMPLE

A. Technical Coefficients and Constraints

A large series of production plans can be run to investigate the effect of changing management and biological parameters. For the purposes of illustration only, three production plans were considered in this paper, corresponding to purebreds with mature cow sizes of 400 kg, 450 kg and 500 kg, and here referred to as small, medium and large.

The emphasis was on beef cattle characteristics, so beef was the only livestock enterprise. Cropping activities were run solely for the provision of animal feed. Activities involving the sale of crops were therefore fixed in the solution at a level of zero.

Four types of resource constraints were defined: labor, land, crop storage capacity and cow herd size. Hired labor was available

up to a monthly limit of 200 h; no land could be hired above that which was owned (taken as the approximate average for all Ontario of 80 ha (OMAF 1973)); crop storage facilities were available to a maximum of 200 metric tons for barley, 200 metric tons for corn grain, 800 metric tons for corn silage and 300 metric tons for hay. All values refer to capacities of wet material. The cow herd size was fixed at 100 in each production plan.

A single value for viability of 0.95 was used in this example. With this viability and the age distribution described previously, the proportion of cows culled for old age was determined to be 0.0462, the proportion of heifers required for replacement at 7 mo of age to be 0.2585, and the proportion of females entering the feedlot to be 0.2165.

B. Results and Discussion

The results for enterprises represented by the three LP solutions showed a higher total gross margin for large cows than for medium or small cows (Table 4). The gross margin also provided the returns above production costs incurred each year on a per head, a per hectare or a per unit weight basis. Interest payments on working capital

Table 4. Costs and returns of cow-calf feedlot enterprises at three cow sizes

Item	Small (400 kg) (\$)	Medium (450 kg) (%) (relative to small)	Large (500 kg) (%) (relative to small)
Cow size	—	112.5	125.0
Cost of crop production	4,369	110.3	123.8
Cost of purchased feed	523	231.8	364.5
Cost of hired labor	927	114.8	142.5
Variable livestock costs (other than feed, hired labor and interest)	4,850	100.0	100.0
Interest charges†	2,150	112.5	125.2
Total costs	12,819	112.1	126.2
Total cost/metric ton beef	428	99.6	101.0
Returns from culled animals	4,436	112.5	125.0
Returns from feedlot animals	16,962	112.5	125.0
Returns from salvaged dead animals	13	100.0	100.0
Total returns	21,411	112.5	125.0
Total gross margin	8,592	113.1	123.2
Gross margin/cow	85.92	113.1	123.2
Gross margin/ha	126	96.6	105.2
Gross margin/kg cow	0.215	100.5	98.6

†Interest charges were incurred on funds required to meet variable costs and to purchase cattle.

included interest on livestock valuation, thus giving borrowed funds sufficient to have purchased all the cattle at the beginning of the production year. Cattle valuation varied for each enterprise depending on cow sizes in this case, although they would also vary with herd sizes or reproductive rates if these were allowed to change. The land, buildings, machinery and owner labor and management were common to all plans. The three enterprises could thus be compared on the basis of gross margins.

The costs of crop production, purchased feed, hired labor and net interest are compared in Table 4. Variable costs for livestock (other than feed, labor and interest) could also be compared, although in this example they were equal for all plans since herd size was fixed at 100 cows. Optimal production plans varied for the three enterprises, particularly for cropping programs (Table 5). Since land was not a constraint in the small cow enterprise, this plan used as much inexpensive cropping as possible in the form of hay and pasture. Because the cost of barley per Mcal of energy was slightly less than corn grain (Table 1), barley was grown in preference to corn grain for the small and medium cow enterprises. All plans were similar in that no feeds were purchased, other than creep feed where no alternative was available. The cropping and purchasing program might have been different, however, if the production situation had been

changed. For example, an increase in herd size might have been allowed resulting in all available land being used. Sales of feed and grain could also have altered rations and total programs.

Returns are also shown by class of stock (Table 4). Culled and feedlot animals can be seen to give dollar returns proportional to cow size, following from the proportionality of slaughter weights and mature weights. Salvaged dead animals were an exception since constant per head values were assumed.

The higher total gross margins associated with large cows were not reflected in reduced costs per metric ton of beef, where the total weight of beef included culled females in addition to feedlot animals. Gross margins per unit weight of cow or per hectare of land did not necessarily rank in the same order as total gross margins. Comparisons with gross margins may indicate appropriate choices for farm management decisions, although they do not necessarily reflect biological efficiency.

The quantity of resources used varied for each plan. Total gross margins from small cows were limited by the herd size restriction. Consequently, there was incomplete use of land, making it a slack activity (Table 5). Although hired labor also represented a slack activity in all enterprises, more was used for medium and large cows, as required to satisfy the comparatively labor-

Table 5. Crop and meat production at three cow sizes

Item	Small (400 kg) Units as defined in each row	Medium (450 kg) (%) (relative to small)	Large (500 kg) (%) (relative to small)
Cow herd size	100	100.0	100.0
Total land area used (ha)	68.3	117.1	117.1
Meat returns (metric tons)	30.0	112.5	125.0
Total food intake (metric tons DM)	368	112.1	122.0
as % of land area used			
Distribution of land area	Barley	12.5	21.2
	Corn grain	0.0	0.1
	Corn silage	24.1	24.9
	Hay	27.7	28.6
	Pasture	35.7	25.2

intensive cereal cropping program. For the large cow enterprise, dry matter intake was an important constraint on the diet of feedlot cattle, and it was necessary to provide a high energy diet. Therefore, a small proportion of corn grain, which had the highest energy density per kg of dry matter in Table 1, substituted for barley in the large cow enterprise. Although the appetite limit or dry matter intake constraint was most important for feedlot animals in the large cow enterprise, quality of roughage and cereals in the diet as related to growth rates was sufficiently important in all three plans to result in appetite being a limiting factor for all feedlot animals. Dry matter intakes for breeding cows only reached a maximum in summer, during lactation. This constraint only applied to small cows, because the other two enterprises used better quality diets for cows as a result of the constraint on land area.

The net result of such constraints on appetite and quality of diet is that a comparison of annual feed requirements as in Table 5 is complicated by metabolic body size, land area constraints and the relative energy and protein densities of different dietary components. Medium and large enterprises required more annual feed on a dry matter weight basis (+12.1 and +22.0%, respectively).

The constraint on herd size was important in all cases. Another cow would have allowed an extra \$49 total gross margin for small cows, \$54 for medium cows and \$38 for large cows, as shown by the marginal revenue (LP shadow price) given in the optimal solution. This shows that with the optimal allocation of resources a unit change in herd size would be most beneficial for medium cows, under the conditions defined in the model. With an increase in the limiting resource, the shadow price of a different cow size might be highest. Results must therefore be given a dynamic interpretation by comparing many solutions where various resources are limiting, each at various levels. Another hectare of land would have provided an extra \$8 total gross margin for medium cows and \$22 for large cows. Because land rental charges were \$60 per hectare, additional land would not be used even

if the restriction on rented land were removed, other factors remaining constant (particularly herd size).

In summary, the results describe the application of the LP model to three enterprises varying in cow size, but fixed for such factors as herd size, farm size, culling rate, viability and beef and feed prices. These data are therefore illustrative only and are not intended to give any definitive answers to the importance of cow size to profitability. Within the constraints of this example, total gross margins for each enterprise rank in the same order as cow size. Gross margins per unit weight of output (per metric ton of beef) or per unit weight of animal resources (per kg cow) were, however, almost constant for each cow size. A more detailed discussion of cow size is given by Morris and Wilton (1974, unpublished).

IV. CONCLUSIONS

Although linear programming as a technique has been available for a considerable time, its use in animal production has usually been for least-cost ration formulation. The aim of the present paper has been to extend the LP approach to integrated beef cattle production including crops, cow-calf operation and feedlot. A statement of the assumptions in LP, most of which are implicit in other deterministic models, was included to point out the need for clarity in interpreting results of modelling. A set of animal and crop production values was described, along with a statement of the biological assumptions. The need for more detailed information on both biological and economic parameters became apparent. The results of three enterprises with different cow sizes, obtained by finding an LP solution for each enterprise separately, were presented and discussed briefly. A detailed discussion on cow size requires many more LP runs, changing production and management variables in turn. Because of the positive genetic correlation between mature weight and postweaning growth rate, further information on the profitability of cows of different sizes is urgently needed to complement work, already in progress in the field, on beef performance testing and selection programs and on breed comparisons. Extension of this work to consider dif-

ferent breeds and breed combinations is of considerable interest in view of current import and sales programs for the "exotic" breeds of cattle. Identification of the production and management variables to which the LP solutions are most sensitive is important to indicate the priority areas for improvement at the farm level and for further research.

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